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Radiating Fluid Distribution Interacting with Scalar Field

Koijam Manihar Singh[1]

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Taking the combined energy-momentum tensor for a perfect fluid, radially expanding the radiation and zero-mass scalar field, we investigate their interaction and obtain five new analytic solutions in a spherically symmetric Einstein universe. For the corresponding models various physical and geometrical properties are discussed. In one case an interesting equation of state is derived.

1. INTRODUCTION

Objects with large energy output, either in the form of photons or neutrinos or both in some phases of their evolution, are very much known to exist. Moreover, it is well known that a nonstatic distribution would be radiating energy and so it would be surrounded by an ever-expanding zone of radiation. The early universe was an undifferentiated soup of matter and radiation in a state of thermal equilibrium. During the photon decoupling stage part of the electromagnetic radiation behaved as a perfect fluid comoving with matter, while part behaved like a unidirectional stream moving with fundamental velocity. The discovery of quasistellar objects and their huge energy requirements motivated various authors to develop a theory of hot, convective, supermassive stars where general relativistic effects are important. Einstein showed that the linearized equations of gravitational theory revealed the existence of gravitational radiation. The energy-momentum tensor for radially expanding radiation was first derived by Tolman (1934). Later, Vaidya (1951a,b) generalized it to curvilinear coordinates and using it obtained analytic solutions for radiating fluid spheres in general relativity. Bayin (1978, 1979) studied the field equations for a perfect fluid with a radially expanding radiation. Herrera *et al.* (1980)

[1]Department of Mathematics, Manipur University, Canchipur, Imphal-795003, Manipur, India.

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obtained models of nonstatic radiating fluid spheres. Heller (1982) studied the transition from the dust era to the radiation era (backward in time) in terms of precise, formal models for the dust and radiation within the standard cosmological model.

On the other hand, the concept of scalar fields was introduced by Dirac (1938) in trying to explore Mach's principle, and thereby obtained a theory in which the gravitational constant is no longer a constant, but depends on time. Following this concept, Das (1962), Hyde (1963), Das and Agarwal (1974), and Gurses (1977) obtained solutions for the coupled gravitational and scalar fields. Scalar fields, as they help in explaining the creation of matter in cosmological theories, represent matter fields with spinless quanta. The study of the scalar meson field in general relativity has been initiated to provide an understanding of the nature of space-time and the gravitational field associated with neutral elementary particles of zero spin. Banerjee and Santosh (1981), Froyland (1982), and Accioly *et al.* (1984) studied the interactions of gravitational and scalar fields.

But less work has been done on studying the interaction among a perfect fluid, a field of radially expanding radiation, and a zero-mass scalar field. With this in view, I consider five different cases. The first three are restricted by the equation of state $p = \varepsilon\rho$ by solving different values to ε in each case; one of them turns out to be that of the dust distribution. In most of the cases the models are found to be expanding ones. I also study their other properties, such as redshift, particle horizon, and reality conditions, and try to evaluate the behavior of the scalar field and the radiation field at different ages of the universe in general and of a star in particular.

2. FIELD EQUATIONS

We consider the spherically symmetric line element

$$ds^2 = \exp(\nu)\, dt^2 - \exp(\beta)\, dr^2 - r^2\, d\theta^2 - r^2 \sin^2\theta\, d\varphi^2 \tag{1}$$

where β and ν are functions of r and t.

The energy-momentum tensor for a radiating perfect fluid distribution interacting with a zero-mass scalar field is given by

$$T_{ij} = Z_{ij} + E_{ij} + S_{ij} \tag{2}$$

where Z_{ij} corresponds to the mechanical part of the energy-momentum tensor due to matter and can be taken as the energy-momentum tensor for perfect fluid, so that

$$Z_{ij} = (p + \rho)\, U_i U_j - p g_{ij} \tag{3}$$

where p is the isotropic pressure, ρ is the fluid density and u_i is the four-velocity vector, which satisfies the relation

$$u_i u^i = 1 \tag{4}$$

E_{ij} corresponds to the energy-momentum tensor for spherically symmetric, radially expanding radiation (Vaidya, 1951a,b) and is given by

$$E_{ij} = \sigma \omega_i \omega_j \tag{5}$$

along with

$$\omega^i \omega_i = 0 \tag{6}$$

$$\omega^i_{;j} \omega^j = 0 \tag{7}$$

where σ is the density of the flowing radiation.

S_{ij} corresponds to the energy-momentum tensor for the zero-mass scalar field and is given by

$$S_{ij} = \varphi_i \varphi_j - \tfrac{1}{2} g_{ij} \varphi_k \varphi^k \tag{8}$$

where the scalar potential φ satisfies the Klein–Gordon equation

$$g^{ij} \varphi_{;ij} = 0 \tag{9}$$

Hence, finally, T_{ij} can be written as

$$T_{ij} = (p+\rho) u_i u_j - p g_{ij} + \sigma \omega_i \omega_j + \varphi_i \varphi_j - \tfrac{1}{2} g_{ij} \varphi_k \varphi^k \tag{10}$$

Here we assume comoving coordinate system.
Then

$$u^1 = u^2 = u^3, \qquad u^4 = \exp(-\nu/2) \tag{11}$$

As for notation, anywhere in this problem, a prime and an overdot denote differentiation with respect to r and with respect to t, respectively, and a semicolon followed by a subscript denotes covariant differentiation.

Now, for the metric (1), the Einstein field equation yields

$$\left(\frac{\nu'}{r} + \frac{1}{r^2}\right) e^{-\beta} - \frac{1}{r^2} = 8\pi p - 8\pi\sigma\omega_1\omega^1 + e^{-\beta}\varphi'^2 + e^{-\nu}\dot{\varphi}^2 \tag{12}$$

$$\left(\frac{\beta'\nu'}{4} + \frac{\beta'}{2r} + \frac{\nu'}{2r} - \frac{\nu''}{2} - \frac{\nu'^2}{4}\right) e^{-\beta} + \left(\frac{\ddot{\beta}}{2} + \frac{\dot{\beta}^2}{4} - \frac{\dot{\beta}\dot{\nu}}{4}\right) e^{-\nu} = -8\pi p + e^{-\beta}\varphi'^2 - e^{-\nu}\dot{\varphi}^2 \tag{13}$$

$$\left(\frac{\beta'}{r} - \frac{1}{r^2}\right) e^{-\beta} + \frac{1}{r^2} = 8\pi\rho + 8\pi\sigma\omega_4\omega^4 + e^{-\beta}\varphi'^2 + e^{-\nu}\dot{\varphi}^2 \tag{14}$$

$$\frac{\dot{\beta}}{r} e^{-\nu} = 8\pi\left(\sigma\omega_1\omega^4 + \frac{1}{4\pi}\varphi'\dot{\varphi}e^{-\nu}\right) \tag{15}$$

Again from (6) and (7) we get, respectively,

$$\omega^4 = \omega^1 \exp \frac{\beta - \nu}{2} \tag{16}$$

and

$$\frac{\partial \omega^1}{\partial r} + \exp\left(\frac{\beta - \nu}{2}\right) \frac{\partial \omega^1}{\partial t} + \omega^1 \left[\frac{1}{2}(\beta' + \nu') + \dot{\beta} \exp\left(\frac{\beta - \nu}{2}\right)\right] = 0 \tag{17}$$

Also from (9) we have

$$e^{-\nu}\ddot{\varphi} - e^{-\beta}\varphi'' + \frac{1}{2}\left(\beta' - \nu' - \frac{4}{r}\right) e^{-\beta}\varphi' + \frac{1}{2}(\dot{\beta} - \dot{\nu}) e^{-\nu}\dot{\varphi} = 0 \tag{18}$$

3. SOLUTIONS OF THE FIELD EQUATIONS

3.1. Case I

Here we take up the case $p = \rho$. Subtracting (12) from (14), we get

$$\frac{1}{r}[1 - e^{-\beta}] = \frac{1}{2}(\beta' - \nu') \tag{19}$$

the solution of which is

$$\begin{aligned} \beta &= \log\left(\frac{b_0}{r} + a_0\right) \\ \nu &= (2a_0 - 3)\log r + \log(a_0 r + b_0) - \frac{2b_0}{r} + c_0 \end{aligned} \tag{20}$$

where a_0, b_0 and c_0 are arbitrary constants.

Subsequently, equations (16)-(18), respectively, reduce to the forms

$$\omega^4 = r^{1-a_0}\omega^1 \exp\left(\frac{b_0}{r} - \frac{c_0}{2}\right) \tag{21}$$

$$\frac{\partial \omega^1}{\partial r} + r^{1-a_0} \exp\left(\frac{b_0}{r} - \frac{c_0}{2}\right) \frac{\partial \omega^1}{\partial t} + \left(\frac{a_0}{a_0 r + b_0} + \frac{a_0 - 2}{r} + \frac{b_0}{r^2}\right)\omega^1 = 0 \tag{22}$$

$$r^{3-2a_0} \exp\left(\frac{2b_0}{r} - c_0\right)\ddot{\varphi} - r\varphi'' - \left(1 + a_0 + \frac{b_0}{r}\right)\varphi' = 0 \tag{23}$$

Here (23) gives

$$\varphi = \int r^{-(1+a_0)} \exp(b_0/r + a_1)\, dr + a_2 t + a_3 \tag{24}$$

where a_1, a_2, and a_3 are arbitrary constants. Again the solution of (22) is

$$\omega^1 = b_1(a_0 r + b_0)^{-1} r^{-(a_0-2)} \exp(b_0/r) \tag{25}$$

where b_1 is an arbitrary constant. Therefore from (21) we get

$$\omega^4 = b_1(a_0 r + b_0)^{-1} r^{-(2a_0-3)} \exp(2b_0/r - c_0/2) \tag{26}$$

Thus, (15) gives

$$\sigma = \frac{1}{4\pi} a_2 b_1^2 r^{-2} \exp\left(a_1 - \frac{c_0}{2}\right) \tag{27}$$

Also from (13) we have

$$\begin{aligned} p = \frac{1}{8\pi}\left(\frac{r}{a_0 r + b_0}\right)&\left\{\frac{1}{4}\left\{\left(\frac{2a_0-3}{r} + \frac{a_0}{a_0 r + b_0} + \frac{2b_0}{r^2}\right)^2 + \frac{1}{2r}\left(\frac{1}{r} + \frac{a_0}{a_0 r + b_0} + \frac{2b_0}{r^2}\right)\right.\right. \\ &+ \frac{b_0}{4r(a_0 r + b_0)}\left(\frac{2a_0-3}{r} + \frac{a_0}{a_0 r + b_0} + \frac{2b_0}{r^2}\right) \\ &\left.\left. - \left[\frac{2a_0-3}{2r^2} + \frac{a_0^2}{2(a_0 r + b_0)^2} + \frac{4b_0}{r^3}\right]\right]\right\} \\ &+ \frac{1}{8\pi}(a_0 r + b_0)^{-1} r^{3-2a_0}[r^{-4}\exp(2a_1) - a_2^2 \exp(-c_0)] \exp\frac{2b_0}{r} \end{aligned} \tag{28}$$

3.2. Case II

In this case we take $\rho = 3p$. Let us consider the scalar field to be of the form

$$\varphi = x_1(t) + q_1(r) \tag{29}$$

Then (18) reduces to the form

$$\left[\ddot{x}_1 + \frac{1}{2}(\dot{\beta} - \dot{\nu})\dot{x}_1\right]\exp(-\nu) - \left[q_1'' - \frac{q_1'}{2}\left(\beta' - \nu' - \frac{4}{r}\right)\right]\exp(-\beta) = 0$$

which gives

$$\dot{\varphi} = \dot{x}_1 = c \exp\frac{\nu-\beta}{2}, \qquad \varphi' = q_1' = \frac{c}{r^2}\exp\frac{\nu-\beta}{2} \tag{30}$$

where c is an arbitrary constant.

Now from (12) and (13) we get

$$\begin{aligned} &\frac{\nu''}{2} + \frac{\nu'^2}{4} - \frac{\beta'\nu'}{4} - \frac{\beta'}{2r} - \frac{\nu'}{2r} - \left(\frac{\ddot{\beta}}{2} + \frac{\dot{\beta}^2}{4} - \frac{\dot{\beta}\dot{\nu}}{4}\right)\exp(\beta - \nu) \\ &+ \frac{1}{r^2}[\exp(\beta) - 1] = 8\pi\sigma\omega_1\omega^1 \exp(\beta) - 2c^2 r^{-4}\exp(\beta - \nu) \end{aligned} \tag{31}$$

Again from (15) and (16) we have

$$8\pi\sigma\omega_1\omega^1\exp(\beta)=\frac{\dot{\beta}}{r}\exp\left(\frac{\beta-\nu}{2}\right)-2c^2r^{-2}\exp\left(\frac{\beta-\nu}{2}\right) \tag{32}$$

Equations (31) and (32) give

$$\frac{\nu''}{2}+\frac{\nu'^2}{4}-\frac{\beta'\nu'}{4}-\frac{\beta'}{2r}-\frac{\nu'}{2r}+[\exp(\beta)-1]r^{-2}-\left(\frac{\ddot{\beta}}{2}+\frac{\dot{\beta}^2}{4}-\frac{\dot{\beta}\dot{\nu}}{4}\right)\exp(\beta-\nu)$$
$$=\frac{\dot{\beta}}{r}\exp\left(\frac{\beta-\nu}{2}\right)-2c^2r^{-2}\exp\left(\frac{\beta-\nu}{2}\right)-2c^2r^{-4}\exp(\beta-\nu) \tag{33}$$

Again from (12) and (14) we get

$$\frac{\beta'-\nu'}{r}-\frac{2}{r^2}[1-\exp(\beta)]=16\pi p\exp(\beta) \tag{34}$$

Now using this relation in (13), we have

$$\frac{\nu''}{2}+\frac{\nu'^2}{4}-\frac{\beta'\nu'}{4}-\frac{(\beta'-\nu')}{r}+[1-\exp(\beta)]r^{-2}$$
$$+c^2[r^{-4}\exp(\beta-\nu)-1]-\left(\frac{\ddot{\beta}}{2}+\frac{\dot{\beta}^2}{4}-\frac{\dot{\beta}\dot{\nu}}{4}\right)\exp(\beta-\nu)=0 \tag{35}$$

Adding (35) and (33), we get

$$\nu''+\frac{\nu'^2}{2}-\frac{\beta'\nu'}{2}-\frac{3\beta'-\nu'}{2r}+3c^2r^{-4}\exp(\beta-\nu)-\frac{\dot{\beta}}{r}\exp\left(\frac{\beta-\nu}{2}\right)-c^2$$
$$-\left(\ddot{\beta}+\frac{\dot{\beta}^2}{2}-\frac{\dot{\beta}\dot{\nu}}{2}\right)\exp(\beta-\nu)+2c^2r^{-2}\exp\left(\frac{\beta-\nu}{2}\right)=0 \tag{36}$$

Here we make the substitution

$$\exp(\beta)=A_0\psi^n(r)g^2(t)$$
$$\exp(\nu)=B_0\psi^{-m}(r)k^2(t) \tag{37}$$

where A_0 and B_0 are arbitrary constants.

Consequently, (36) transforms into

$$\frac{m}{2}\left(\frac{m}{2+1}\right)\psi^{-2}\psi'^2-\frac{m}{2}\psi^{-1}\psi''+\frac{mn}{4}\psi^{-2}\psi'^2+\frac{3c^2A_0}{2B_0}\frac{g^2}{k^2}r^{-4}\psi^{m+n}$$
$$-\frac{1}{2r}\left(\frac{3n}{2}\psi^{-1}\psi'+\frac{m}{2}\psi^{-1}\psi'\right)-\left(\frac{A_0}{B_0}\right)^{1/2}\frac{\dot{g}}{rk}\psi^{m/2+n/2}-\frac{c^2}{2}$$
$$+\left(\frac{A_0}{B_0}\right)^{1/2}\frac{c^2g}{k}r^{-2}\psi^{(m+n)/2}-\frac{A_0}{B_0}\frac{g^2}{k^2}\left(\frac{\ddot{g}}{g}-\frac{\dot{k}\dot{g}}{kg}\right)\psi^{m+n}=0 \tag{38}$$

Since this equation is highly nonlinear, we assume

$$g(t)=s_0k(t) \tag{39}$$

where s_0 is an arbitrary constant. Then we get

$$\left[\frac{m}{2}\left(\frac{m}{2+1}\right)+\frac{mn}{4}\right]\psi^{-2}\psi'^2-\frac{m}{2}\psi^{-1}\psi''-\frac{m+3n}{3r}\psi^{-1}\psi' \\ +\frac{3c^2s_0^2A_0}{2B_0}r^{-4}\psi^{m+n}-\frac{c^2}{2}-\frac{s_0^2A_0}{B_0}\left(\frac{\ddot{g}}{g}-\frac{\dot{g}^2}{g^2}\right)\psi^{m+n} \\ -\left(\frac{A_0}{B_0}\right)^{1/2}\frac{\dot{g}}{g}\frac{s_0}{r}\psi^{(m+n)/2}+\left(\frac{A_0}{B_0}\right)^{1/2}c^2s_0r^{-2}\psi^{(m+n)/2}=0 \tag{40}$$

We further assume

$$\dot{k}(t)=zk(t) \tag{41}$$

where z is a constant. Thus,

$$k(t)=\exp(c_1+zt) \tag{42}$$

where c_1 is an arbitrary constant. Then (40) becomes

$$\left[\frac{m}{2}\left(\frac{m}{2}+1\right)+\frac{mn}{4}\right]\psi^{-2}\psi'^2-\frac{m+3n}{4r}\psi^{-1}\psi'-\frac{c^2}{2}-\frac{m}{2}\psi^{-1}\psi'' \\ +\frac{3c^2s_0^2A_0}{2B_0}r^{-4}\psi^{m+n}-\left(\frac{A_0}{B_0}\right)^{1/2}\frac{zs_0}{r}\psi^{(m+n)/2} \\ +\left(\frac{A_0}{B_0}\right)^{1/2}c^2s_0r^{-2}\psi^{(m+n)/2}=0 \tag{43}$$

Now we take up the case

$$m=-n \tag{44}$$

Then (43) becomes

$$\frac{\psi''}{\psi}-\left(\frac{\psi'}{\psi}\right)^2-r^{-1}\frac{\psi'}{\psi} \\ =\frac{3c^2s_0^2A_0}{mB_0}r^{-4}+\left(\frac{A_0}{B_0}\right)^{1/2}\frac{2c^2s_0}{m}r^{-2}-\left(\frac{A_0}{B_0}\right)^{1/2}\frac{2zs_0}{m}r^{-1}-\frac{c^2}{m}$$

which, on integrating twice, gives

$$\log\psi=\frac{c_1}{2}r^2-\frac{c^2}{2m}\left(\log r-\frac{1}{2}\right)r^2+\frac{2zA_0s_0}{mB_0}r \\ -\frac{c^2s_0}{m}\left(\frac{A_0}{B_0}\right)^{1/2}\log r+\frac{3c^2s_0^2A_0}{8mB_0}r^{-2}+c_2 \tag{45}$$

where c_1 and c_2 are constants of integration.

Next let us consider, without loss of generality,

$$A_0 = B_0 = c = 1 \tag{46}$$

Then, from (45) we have

$$\psi = \exp\left[\frac{3s_0^2}{8m} r^{-2} + \frac{2zs_0}{m} r + \frac{1}{2}\left(c_1 + \frac{1}{2m}\right) r^2 + c_2\right] + r^{-(s_0/m + r^2/2m)} \tag{47}$$

Using (47) in (37), we get

$$\beta = 2zt - m \log\left\{\exp\left[\frac{1}{2}\left(c_1 + \frac{1}{2m}\right) r^2 + \frac{2zs_0}{m} r + \frac{3s_0^2}{8m} r^{-2} + c_2\right]\right.$$
$$\left. + r^{-(s_0/m + r^2/2m)}\right\} + 2c_1 + \log(A_0 S_0^2) \tag{48}$$

$$\nu = 2zt - m \log\left\{\exp\left[\frac{1}{2}\left(c_1 + \frac{1}{2m}\right) r^2 + \frac{2zs_0}{m} r + \frac{3s_0^2}{8m} r^{-2} + c_2\right]\right.$$
$$\left. + r^{-(s_0/m + r^2/2m)}\right\} + 2c_1 + \log B_0 \tag{49}$$

Thus, from (34) we have

$$p = \frac{1}{8\pi} r^{-2} - \frac{s_0^{-2} r^{-2}}{8\pi A_0}\left\{\exp\left[\frac{1}{2}\left(c_1 + \frac{1}{2m}\right) r^2 + \frac{2zs_0}{m} r\right.\right.$$
$$\left.\left. + \frac{3s_0^2}{8m} r^{-2} + c_2\right] + r^{-(s_0/m + r^2/2m)}\right\}^m \exp(-2zt - 2c_1) \tag{50}$$

Also (17) gives

$$\omega^1 = c_3\left\{\exp\left[\frac{1}{2}\left(c_1 + \frac{1}{2m}\right) r^2 + \frac{2zs_0}{m} r + \frac{3s_0^2}{8m} r^{-2} + c_2\right]\right.$$
$$\left. + r^{-(s_0/m + r^2/2m)}\right\}^m - 2zt \tag{51}$$

where c_3 is an arbitrary constant.

Therefore, from (16) we get

$$\omega^4 = \left(\frac{A_0}{B_0}\right)^{1/2} s_0\left(c_3\left\{\exp\left[\frac{1}{2}\left(c_1 + \frac{1}{2m}\right) r^2 + \frac{2zs_0}{m} r + \frac{3s_0^2}{8m} r^{-1} + c_2\right]\right.\right.$$
$$\left.\left. + r^{-(s_0/m + r^2/2m)}\right\}^m - 2zt\right) \tag{52}$$

Again (18) gives

$$e^{-\nu}\ddot{\varphi}-e^{-\beta}\varphi''-(2/r)e^{-\beta}\varphi'=0$$

the solution of which is

$$\varphi=-d_1t+d_2r^{-1}+d_3 \tag{53}$$

where d_1, d_2, and d_3 are arbitrary constants.

From (15) we have

$$(\dot{\beta}/r)e^{-\nu}-2\,e^{-\nu}\varphi'\dot{\varphi}=8\pi\sigma\omega_1\omega^4$$

which gives

$$\sigma=\frac{\bar{A}_0^{3/2}\bar{B}_0^{1/2}\bar{S}_0^3}{4\pi}(d_1d_2r^{-2}-zr^{-1})\left\{r^{-(1/a)(r^2/2+s_0)}\right.$$
$$\left.+\exp\left[\frac{1}{2}\left(c_1+\frac{1}{2m}\right)\frac{r^2}{2}+\frac{2zs_0}{a}r+\frac{3s_0^2}{8a}r^{-2}+c_2\right]\right\}^{2a}$$
$$\times\left(c_3\left\{r^{-(1/a)(r^2/2+s_0)}+\exp\left[\left(\frac{c_1}{2}+\frac{1}{4a}+\frac{2zs_0}{a}\right)r+c_2\right.\right.\right.$$
$$\left.\left.\left.+\left(\frac{3s_0^2}{8a}\right)r^{-2}\right]\right\}^a-2zt\right)^{-2}\exp(-4zt-4c_1) \tag{54}$$

3.3. Case III

For this case we take $\varepsilon=0$ in the equation of state

$$p=\varepsilon\rho$$

Now in the same manner as in case II we get

$$\dot{\varphi}=c\exp\frac{\nu-\beta}{2},\qquad \varphi'=cr^{-2}\exp\frac{\beta-\nu}{2}$$

where c is an arbitrary constant.

Then equations (12)-(15), respectively, become

$$r^{-1}\nu'+r^{-2}(1-e^{\beta})=-8\pi\sigma\omega_1\omega^1e^{\beta}+c^2(1+r^{-4}e^{\beta-\nu}) \tag{55}$$

$$\frac{\nu''}{2}+\frac{\nu'^2}{4}-\frac{\beta'\nu'}{4}-\frac{r^{-1}(\beta'-\nu')}{2}-\left(\frac{\ddot{\beta}}{2}+\frac{\dot{\beta}^2}{4}-\frac{\dot{\beta}\dot{\nu}}{4}\right)e^{\beta-\nu}=c^2-c^2r^{-4}\,e^{\beta-\nu} \tag{56}$$

$$r^{-1}\beta'+r^{-2}(e^{\beta}-1)=8\pi\rho\,e^{\beta}+8\pi\sigma\omega_4\omega^4\,e^{\beta}+c^2(1+r^{-4}e^{\beta-\nu}) \tag{57}$$

and

$$r^{-1}\dot{\beta}=8\pi\sigma\omega_1\omega^4\,e^{\nu}+2c^2r^{-2} \tag{58}$$

Now if we use the substitution

$$e^{\beta}=f^2(r)g^2(t), \qquad e^{\nu}=h^2(r)k^2(t) \tag{59}$$

in (56) we get

$$\frac{h''}{h}-\frac{f'h'}{fh}-\left(\frac{f'}{f}-\frac{h'}{h}\right)r^{-1}-\frac{f^2g^2}{h^2k^2}\left(\frac{\ddot{g}}{g}-\frac{\dot{g}\dot{k}}{gk}\right)+c^2\left(\frac{f^2g^2}{h^2k^2}\right)r^{-4}-c^2=0 \tag{60}$$

Since this equation involves four unknowns, to simplify it we assume a relation

$$g(t)=s_1k(t) \tag{61}$$

where s_1 is an arbitrary constant. Then (60) becomes

$$\frac{h''}{h}-\frac{f'h'}{fh}-\left(\frac{f'}{f}-\frac{h'}{h}\right)r^{-1}-\frac{s_1^2f^2}{h^2}\left(\frac{\ddot{g}}{g}-\frac{\dot{g}^2}{g^2}\right)+c^2s_1^2\frac{f^2}{h^2}r^{-4}-c^2=0 \tag{62}$$

Next, assuming another relation

$$\dot{g}=z_1g \tag{63}$$

where z_1 is a constant, (62) transforms into

$$\frac{h''}{h}-\frac{f'h'}{fh}-\left(\frac{f'}{f}-\frac{h'}{h}\right)r^{-1}+c^2s_1^2\frac{f^2}{h^2}r^{-4}-c^2=0 \tag{64}$$

Also, from (61) and (63) we get

$$k=\exp(z_1t+c_2) \tag{65}$$

where c_2 is a constant of integration. Now, to solve (64), we need one more relation between f and h, which we take in the form

$$f^2=A_0\psi^n(r), \qquad h^2=B_0\psi^{-m}(r) \tag{66}$$

where A_0 and B_0 are arbitrary constants.

Then (64) becomes

$$\psi^{-1}\psi''-\left(\frac{m+n}{2}+1\right)\psi^{-2}\psi'^2-\frac{2A_0c^2s_1^2}{mB_0}r^{-4}\psi^{m+n}$$
$$+\frac{m+n}{m}r^{-1}\psi^{-1}\psi'+\frac{2c^2}{m}=0 \tag{67}$$

Now we take up the case

$$m=-n \tag{68}$$

Consequently, (67) takes the form

$$\psi^{-1}\psi''-\psi^{-2}\psi'^2=\frac{2A_0c^2s_1^2}{mB_0}r^{-4}-\frac{2c^2}{m}$$

which gives

$$\psi=y_2\exp\left(\frac{A_0c^2s_1^2}{3mB_0}r^{-2}+y_1r-\frac{c^2}{m}r^2\right) \tag{69}$$

where y_1 and y_2 are constants of integration.

Thus, we get

$$\beta=d_0+2z_1t+c^2r^2-my_1r-\frac{A_0c^2s_1^2}{3B_0}r^{-2} \tag{70}$$

$$\nu=d+2z_1t+c^2r^2-my_1r-\frac{A_0c^2s_1^2}{3B_0}r^{-2} \tag{71}$$

where

$$d_0=\log(A_0s_1^2y_2^{-mm})+c_2, \qquad d=\log(B_0y_2^{-m})+c_2$$

Now, from (17) we have [using (70) and (71)]

$$\frac{\partial\omega^1}{\partial r}+\exp\left(\frac{d_0-d_1}{2}\right)\frac{\partial\omega^1}{\partial t}$$
$$+\left[2c^2r+\frac{2A_0c^2s_1^2}{3B_0}r^{-3}-my_1+2z_1\exp\left(\frac{d_0-d}{2}\right)\right]\omega^1=0$$

the solution of which is

$$\omega^1=\exp\left[\frac{A_0c^2s_1^2}{3B_0}r^{-2}+my_1r-2z_1\left(\exp\frac{d_0-d}{2}\right)r-c^2r^2\right] \tag{72}$$

Therefore, from (16) we get

$$\omega^4=\exp\left[\frac{c^2s_1^2A_0}{3B_0}r^{-2}+(1-2z_1r)\exp\left(\frac{d_0-d}{2}\right)+my_1r-c^2r^2\right] \tag{73}$$

Again using (70) and (71) in (18), we get

$$e^{-\nu}\ddot{\varphi}-e^{-\beta}\varphi''-2r^{-1}e^{-\beta}\varphi'=0$$

the solution of which is

$$\varphi=d_4r^{-1}d_5-d_6t \tag{74}$$

where d_4, d_5, and d_6 are arbitrary constants.

Also subtracting (55) from (57), we have

$$r^{-1}(\beta'-\nu')-2r^{-2}+2r^{-2}e^{\beta}=8\pi\rho\, e^{\beta}$$

from which we get

$$\rho=\frac{1}{4\pi r^2}\left[1-\exp\left(\frac{A_0c^2s_1^2}{3B_0}r^{-2}+my_1r-c^2r^2-2z_1t-d_0\right)\right] \tag{75}$$

Again from (62) we have

$$\sigma=\frac{1}{8\pi}(2c^2r^{-2}-2z_1r^{-1})\exp\left[-(4z_1r+1)\exp\left(\frac{d_0-d}{2}\right)-4z_1t-d-d_0\right] \tag{76}$$

3.4. Case IV

From (12) and (13) we have

$$\left(\frac{\nu''}{2}+\frac{\nu'^2}{4}-\frac{\beta'\nu'}{4}-\frac{\beta'}{2r}+\frac{\nu'}{2r}-\frac{\nu'}{r}-r^{-2}\right)e^{-\beta}+r^{-2}$$
$$-\left(\frac{\ddot\beta}{2}+\frac{\dot\beta^2}{4}-\frac{\dot\beta\dot\nu}{4}\right)e^{-\nu}=8\pi\sigma\omega_1\omega^1-2\,e^{-\beta}\varphi'^2 \tag{77}$$

Also from (14) and (15) we get

$$8\pi\sigma\omega_1\omega^1=\frac{\dot\beta}{r}\exp\left(-\frac{\beta+\nu}{2}\right)-2\varphi'\dot\varphi\exp\left(-\frac{\beta+\nu}{2}\right) \tag{78}$$

Now (77) and (78) give

$$\left(\frac{\nu''}{2}+\frac{\nu'^2}{4}-\frac{\beta'\nu'}{4}-\frac{\beta'}{2r}-\frac{\nu'}{2r}-r^{-2}\right)\exp(-\beta)+\left(\frac{\ddot\beta}{2}+\frac{\dot\beta^2}{4}-\frac{\dot\beta\dot\nu}{4}\right)\exp(-\nu)+r^{-2}$$
$$=\frac{\dot\beta}{r}\exp\left(-\frac{\beta+\nu}{2}\right)-2\varphi'\dot\varphi\exp\left(-\frac{\beta+\nu}{2}\right)-2\varphi'^2\exp(-\beta)$$

a solution of which is

$$\begin{aligned}\beta&=\log(ar^{-1}t^{-1})\\ \nu&=\log(art^{-3})\\ \varphi&=br^{-1/2}t^{-1/2}\end{aligned} \tag{79}$$

where a and b are arbitrary constants connected by

$$a+b^2=0$$

Thus, from (17) we get

$$\frac{\partial\omega^1}{\partial r} r^{-1} t \frac{\partial\omega^1}{\partial t} - r^{-1}\omega^1 = 0$$

which gives

$$\omega^1 = \tfrac{1}{2} q_0 r^{1/2} t^{1/2} \tag{80}$$

where q_0 is an arbitrary constant.

Therefore,

$$\omega^4 = \tfrac{1}{2} q_0 r^{-1/2} t^{3/2} \tag{81}$$

Again (12) and (13) together give

$$p = \frac{a^{-1}}{16\pi}(2r^{-1}t - ar^{-2} - b^2 r^{-2}) \tag{82}$$

From (78) we get

$$\sigma = \frac{1}{4\pi} a^{-2} q_0^{-2} r^{-1}(b^2 r^{-1} + 2t) \tag{83}$$

Also from (14) we have

$$\rho = \frac{1}{8\pi}(2r^{-2} + 3b^{-2} r^{-1} t) \tag{84}$$

3.5. Case V

In this case we take β to be a function of r only. Then, from (15) and (16) we get

$$8\pi\sigma\omega_1\omega^1 \exp\left(\frac{\beta - \nu}{2}\right) + 2\varphi'\dot{\varphi} \exp(-\nu) = 0 \tag{85}$$

Again from (12) and (13) we get

$$\left(-\frac{\nu''}{2} - \frac{\nu'^2}{4} + \frac{\beta'\nu'}{4} + \frac{r^{-1}\beta'}{2} + \frac{r^{-1}\nu'}{2} + r^{-2}\right) e^{-\beta} - r^{-2}$$
$$= 2e^{-\beta}\varphi'^2 - 8\pi\sigma\omega_1\omega^1 \tag{86}$$

Now (85) and (86) give

$$\left(-\frac{\nu''}{2} - \frac{\nu'^2}{4} + \frac{\beta'\nu'}{4} + \frac{r^{-1}}{\beta' 2} + \frac{r^{-1}\nu'}{2} + r^{-2}\right) e^{-\beta} - r^{-2}$$
$$= 2e^{-\beta}\varphi'^2 + 2e^{-(\beta+\nu)/2}\varphi'\dot{\varphi} \tag{87}$$

Here we assume

$$\nu = f_1(r) + g_1(t) \tag{88}$$

$$\varphi = h_1(r) + k_1(t) \tag{89}$$

Then (87) becomes

$$\frac{1}{h'1}\left[\frac{\beta' f_1'}{4} + \frac{r^{-1}\beta'}{2} + \frac{r^{-1}f_1'}{2} + r^{-2} - \frac{f_1''}{2} - \frac{f_1'^2}{2} - \frac{f_1'^2}{4} - r^{-2}\exp(\beta) - 2h_1'^2\right]\exp\left(\frac{f_1-\beta}{2}\right) = 2\dot{k}_1 \exp\left(-\frac{g_1}{2}\right) \tag{90}$$

Since the left-hand side is a function of r only, whereas the right-hand side is a function of t only, we can equate both of them to a constant. Thus, now (90) separates into

$$\frac{1}{h'1}\left[\frac{\beta' f_1'}{4} + \frac{r^{-1}\beta'}{2} + \frac{r^{-1}f_1'}{2} + r^{-2} - \frac{f_1''}{2} - \frac{f_1'^2}{4} - r^{-2}\exp(\beta) - 2h_1'^2\right] \times \exp\left(\frac{f_1-\beta}{2}\right) = c_4 \tag{91}$$

and

$$2\dot{k}_1 \exp(-g_1/2) = c_4 \tag{92}$$

where c_4 is an arbitrary constant.

Now (91) gives as a solution

$$\begin{aligned} \beta &= \log(a_4 r) \\ f_1 &= 3\log(a_4 r) \\ h &= \tfrac{1}{4}(c_4^2 a_4^{-2}+24)^{1/2}\log\{(c_4^2 a_4^{-2}+24)^{1/2} - [(c_4^2 a_4^{-2}+24) - 8a_4 r]^{1/2}\} \\ &\quad - \tfrac{1}{4}c_4 a_4^{-1}\log r - \tfrac{1}{4}(c_4^2 a_4^{-2}+24)^{1/2}\log\{(c_4^2 a_4^{-2}+24)^{1/2} \\ &\quad + [(c_4^2 a_4^{-2}+24) - 8a_4 r]^{1/2}\} + \tfrac{1}{2}[(c_4^2 a_4^{-2}+24) - 8a_4 r]^{1/2} + d_5 \end{aligned} \tag{93}$$

where a_4 and d_5 are arbitrary constants.

Again (92) gives

$$g_1 = -2\log\frac{c_4 t + d_4}{4b_4}, \qquad k_1 = 2b_4\log\frac{c_4 t + d_4}{4b_4} \tag{94}$$

where b_4 and d_4 are arbitrary constants.

Thus, we get

$$\nu = 3\log(a_4 r) - 2\log[\tfrac{1}{4}b_4^{-1}(c_4 t + d_4)] \tag{95}$$

$$\begin{aligned}\varphi &= \tfrac{1}{4}(c_4^2 a_4^{-2}+24)^{1/2}\log\{(c_4^2 a_4^{-2}+24)^{1/2} - [(c_4^2 a_4^{-2}+24) - 8a_4 r]^{1/2}\}\\ &\quad -\tfrac{1}{4}(c_4^2 a_4^{-2}+24)^{1/2}\log\{(c_4^2 a_4^{-2}+24)^{1/2} + [(c_4^2 a_4^{-2}+24) - 8a_4 r]^{1/2}\}\\ &\quad +\tfrac{1}{2}[(c_4^2 a_4^{-2}+24) - 8a_4 r]^{1/2} - \tfrac{1}{4}c_4 a_4^{-1}\log r\\ &\quad + 2b_4\log[\tfrac{1}{4}b_4^{-1}(c_4 t + d_4)] + d_5\end{aligned} \tag{96}$$

Now using (93) and (95) in (17), we have

$$4a_4 b_4 r\frac{\partial\omega^1}{\partial r} + (c_4 t + d_4)\frac{\partial\omega^1}{\partial t} + 8a_4 b_4 \omega^1 = 0$$

the solution of which is

$$\omega^1 = b_2 r^{-2} + b_3(c_4 t + d_4)^{-8a_4 b_4/c_4} \tag{97}$$

where b_2 and b_3 are constants of integration. Therefore, from (16) we get

$$\omega^4 = \tfrac{1}{4}a_4^{-1}b_4^{-1}r^{-1}(c_4 t + d_4)[b_2 r^{-2} + b_3(c_4 t + d_4)^{-8a_4 b_4/c_4}] \tag{98}$$

Also from (85) we have

$$\begin{aligned}\sigma &= \frac{1}{8\pi}c_4(a_4 r)^{-3}[b_2 r^{-2} + b_3(c_4 t + d_4)^{-8a_4 b_4/c_4}]^{-2}\\ &\quad \times\left\{\frac{1}{4r}[(c_4^2 a_4^{-2}+24) - 8a_4 r]^{1/2} - \tfrac{1}{4}c_4 a_4^{-1} r^{-1}\right\}\end{aligned} \tag{99}$$

Again from (12) and (13) we get

$$\begin{aligned}&\left(\frac{\nu''}{2} + \frac{\nu'^2}{4} - \frac{\beta'\nu'}{4} - \frac{1}{2}r^{-1}\beta' + \frac{3}{2}r^{-1}\nu' + r^{-2}\right)e^{-\beta} - r^{-2}\\ &\quad = 16\pi p - 8\pi\sigma\omega_1\omega^1 + 2e^{-\nu}\dot{\varphi}^2\end{aligned}$$

which gives

$$\begin{aligned}p &= \frac{5}{16\pi}a_4^{-1}r^{-3} + \frac{1}{16\pi}c_4(a_4 r)^{-2}\\ &\quad \times\left[\frac{1}{4r}(c_4^2 a_4^{-2} + 24 - 8a_4 r)^{1/2} - \frac{3}{4}c_4 a_4^{-1} r^{-1} - a_4^2 c_4^{-1}\right]\end{aligned} \tag{100}$$

Also from (14) we have

$$\rho = \frac{3}{2}r^{-2} + \frac{3}{8}c_4 a_4^{-2} r^{-3}(c_4^2 a_4^{-2} + 24 - 8a_4 r)^{1/2} - \frac{1}{8}a_4^{-1}(5c_4^2 a_4^{-2} + 12)r^{-3} \tag{101}$$

4. CONCLUSIONS

4.1. Case I

In this case the line element takes the form

$$ds^2 = (a_0 r + b_0) r^{2a_0-3} \exp(c_0 - 2b_0 r^{-1})\, dt^2 - (a_0 r + b_0) r^{-1}(dr^2 + r^2\, d\theta^2 + r^2 \sin^2\theta\, d\varphi^2) \tag{102}$$

where a_0, b_0, and c_0 are arbitrary constants.

Here the fluid pressure (and therefore also the fluid density) comes out to be a decreasing function of r.

The condition to be satisfied for the distribution to be a realistic one is

$$\rho > 0$$

which gives

$$\tfrac{1}{4}[(2a_0-3)r^{-1} + a_0(a_0 r + b_0)^{-1} + 2b_0 r^{-2}]^2 + \frac{r^{-1}}{2}[r^{-1} + a_0(a_0 r + b_0)^{-1} + 2b_0 r^{-2}] + \tfrac{1}{4} b_0 r^{-1}(a_0 r + b_0)^{-1}[(2a_0-3)r^{-1} + a_0(a_0 r + b_0)^{-1} + 2b_0 r^{-2}] + r^{-2(1+a_0)} \exp(2a_1 + 2b_0 r^{-1}) > a_2^2 r^{2-2a_0} \exp(2b_0 r^{-1} - c_0) + \tfrac{1}{2}(2a_0-3)r^{-2} + \tfrac{1}{2}a_0^2(a_0 r + b_0)^{-2} + 4b_0 r^{-3} \tag{103}$$

Here the radiation density σ as well as the components ω^1 and ω^4 of radiation are decreasing functions of r.

The null ray is described by

$$\int_{t_1}^{t_2} dt = \int_{r_1}^{r_2} r^{1-a_0} \exp(b_0 r^{-1} - c_0/2)\, dr \tag{104}$$

Again as a particular case if $b_0 = 0$, we get the scalar potential as

$$\varphi = a_2 t - a_0^{-1} e^{a_1} r^{-a_0} + a_3$$

and in this case the scalar field becomes an increasing function of time and also an increasing function of r (though not appreciably).

Also in this particular case we get

$$p = \frac{1}{8\pi a_0}[e^{2a_1} r^{-2(1+a_0)} - a_2^2 e^{-c_0} r^{2(1-a_0)} + (a_0^2 - 3a_0 + 3) r^{-2}]$$

Thus, for this model star the radius R is given by

$$p(R) = 0$$

that is, by

$$e^{2a_1} R^{-2(1+a_0)} - a_2^2\, e^{-c_0} r^{2(a_0-1)} + (a_0^2 - 3a_0 + 3) R^{-2} = 0 \tag{105}$$

4.2. Case II

In this case the line element comes out to be

$$ds^2 = B_0\left\{r^{-(s_0/m+r^2/2m)}+\exp\left[\left(c_1+\frac{1}{2m}\right)\frac{r^2}{2}+\left(\frac{2zs_0}{m}\right)r\right.\right.$$
$$\left.\left.+\left(\frac{3s_0^2}{8m}\right)r^{-2}+c_2\right]\right\}^{-m}\exp(2c_1-2zt)\,dt^2-A_0s_0^2\left[r^{-(s_0/m+r^2/m)}\right.$$
$$\left.+\exp\left\{\left[\left(c_1+\frac{1}{2m}\right)\frac{r^2}{2}+\left(\frac{2zs_0}{m}\right)r+\left(\frac{3s_0^2}{8m}\right)r^{-2}+c_2\right]\right\}\right]^{-m}$$
$$\times\exp(2c_1-2zt)\,(dr^2+r^2\,d\theta^2+r^2\sin^2\theta\,d\varphi^2) \tag{106}$$

where c_1, c_2, m, s_0, z, A_0, and B_0 are arbitrary constants.

Here the fluid pressure and the fluid density come out to be decreasing functions of r, but they increase with time.

The condition to be satisfied for the distribution to be a realistic one is

$$A_0s_0^2>\left\{r^{-(s_0/m+r^2/2m)}+\exp\left[\left(c_1+\frac{1}{2m}\right)\frac{r^2}{2}+\left(\frac{2zs_0}{m}\right)r\right.\right.$$
$$\left.\left.+\left(\frac{3s_0^2}{8m}\right)r^{-2}+c_2\right]\right\}^{m}\exp(2zt-2c_1) \tag{107}$$

In this case the scalar expansion is given by

$$\theta=\frac{3}{2}\dot{\beta}\exp\left(-\frac{\nu}{2}\right)$$
$$=3zB_0^{-1/2}\left\{r^{-(s_0/m+r^2/2m)}+\exp\left[\left(c_1+\frac{1}{2m}\right)\frac{r^2}{2}+\left(\frac{2zs_0}{m}\right)r\right.\right.$$
$$\left.\left.+\left(\frac{3s_0^2}{8m}\right)r^{-2}+c_2\right]\right\}^{m/2}\exp(-zt-c_1) \tag{108}$$

Since the expansion is positive here, we see that our model is an expanding one, and the rate of expansion decreases with time.

The scalar field comes out to be an increasing function of r and t both.

The components ω^1 and ω^4 of radiation are found to be decreasing functions of r and they also decrease linearly with time.

The spectral shift in wavelength, as measured at the origin, will be

$$(\lambda+\delta\lambda)/\lambda=(1+zs)^{-1} \tag{109}$$

where s is the metric interval between the observer and the particle.

4.3. Case III

In this case the line element comes out to be

$$ds^2 = \exp\left(c^2r^2 - my_1r - \frac{A_0c^2s_1^2}{3B_0}r^{-2} - 2z_1t + d\right)dt^2$$
$$-\exp\left(c^2r^2 - my_1r - \frac{A_0c^2s_1^2}{3B_0}r^{-2} - 2z_1t + d_0\right)$$
$$\times(dr^2 + r^2\,d\theta^2 + r^2\sin^2\theta\,d\varphi^2) \tag{110}$$

where c, d, d_0, m, s_1, y_1, z_1, A_0, and B_0 are arbitrary constants.

Here the fluid density comes out to be a decreasing function of r, but slowly increases with time.

The condition to be satisfied for the distribution to be a realistic one is

$$\exp\left(\frac{A_0c^2s_1^2}{3B_0}r^{-2} + my_1r - c^2r^2 - 2z_1t - d_0\right) < 1 \tag{111}$$

The scalar expansion in this case comes out to be

$$\theta = 3z_1\exp\left(\frac{A_0c^2s_1^2}{6B_0}r^{-2} + \frac{1}{2}my_1r - \frac{c^2}{2}r^2 - z_1t - \frac{d}{2}\right) \tag{112}$$

Thus, we see that our model here is an expanding one, but the rate of expansion decreases exponentially with time until at $t\to\infty$ it totally stops.

Here the scalar field decreases with r as well as with time.

The components ω^1 and ω^4 of radiation are decreasing functions of r, and the density σ of the flowing radiation is a decreasing function of r and t both.

In this case the spectral shift will be

$$(\lambda + \delta\lambda)/\lambda = (1 + z_1s)^{-1} \tag{113}$$

where s is the metric interval between the observer and the particle.

4.4. Case IV

In this case the line element is

$$ds^2 = art^{-3}\,dt^2 - ar^{-1}t^{-1}\,dr^2 - r^2\,d\theta^2 - r^2\sin^2\theta\,d\varphi^2 \tag{114}$$

where a is an arbitrary constant.

Here the fluid pressure p and the fluid density ρ both are decreasing functions of r, but increasing functions of time.

The scalar field decreases with r and t both, until at $t\to\infty$ it almost vanishes.

The radiation density σ is positive throughout, and the rate of emitting radiation increases with time.

For this model we get an equation of state in the form

$$b^2(\rho - p) = (aq_0)^2\sigma \qquad (115)$$

where a, b, and q_0 are arbitrary constants.

For the distribution to be a realistic one the conditions to be satisfied are

$$\text{(I)} \quad p \geq 0, \qquad \text{(II)} \quad \rho > 0, \qquad \text{(III)} \quad \rho \geq p$$

which respectively give

$$2a^{-1}t \geq (1 + a^{-1}b^2)r^{-1} \qquad (116)$$

$$2b^2 + 3rt > 0 \qquad (117)$$

$$2r^{-1} + 3b^{-2}t \geq a^{-1}t \qquad (118)$$

In this case, the spectral shift in wavelength, as measured at the origin, will be

$$(\lambda + \delta\lambda)/\lambda = d_5 t^3/2 \qquad (119)$$

where d_5 is an arbitrary constant.

4.5. Case V

Here the line element comes out to be

$$ds^2 = 16b_4^{-2}(a_4 r)^3(c_4 t + d_4)^{-2}\, dt^2 - a_4 r\, dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta\, d\varphi^2 \qquad (120)$$

where a_4, b_4, c_4, and d_4 are arbitrary constants.

For this model the fluid pressure and the fluid density both are found to be decreasing functions of r. The radius R of the model star is given by

$$r - \tfrac{1}{4}c_4 a_4^{-2} R^{1/2}[(c_4^2 a_4^{-2} + 24)R^{-1} - 8a_4]^{1/2} + \tfrac{3}{4}c_4^2 a_4^{-3} - 5a_4^{-1} = 0 \qquad (121)$$

which is obtained from $p(R) = 0$. If $c_4 = 0$, we get a static model, for which the radius is $5a_4^{-1}$.

Here the expansion factor θ is found to be zero, reminding us of the steady state model.

For the distribution to be a realistic one we must have, as in case IV,

$$a_4 c_4[(c_4^2 a_4^{-2} + 24)r^{-1} - 8a_4]^{1/2} r^{1/2} - 4a_4^3 r + 20a_4^2 - 3c_4^2 \geq 0 \qquad (122)$$

$$12a_4 r + 3c_4 a_4^{-1} r^{1/2}[(c_4^2 a_4^{-2} + 24)r^{-1} - 8a_4]^{1/2} - 5c_4^2 a_4^{-2} - 12 > 0 \qquad (123)$$

and

$$8a_4^2 r + c_4 r^{1/2}[(c_4^2 a_4^{-2} + 24)r^{-1} - 8a_4]^{1/2} - c_4^2 a_4^{-1} - 16a_4 \geq 0 \qquad (124)$$

Here the scalar field is a decreasing function of r, but it increases with time. The radiation field is also found to be a decreasing function of r.

For this model the spectral shift is

$$(\lambda+\delta\lambda)/\lambda = c_5(c_4 t + d_4) \tag{125}$$

where c_5 is an arbitrary constant.

REFERENCES

Accioly, A. J., Vaidya, A. N., and Som, M. M. (1984). *Nuovo Cimento B* **81**, 235.

Banerjee, A., and Santosh, N. O. (1981). *Journal of Mathematical Physics*, **22**, 1075.

Bayin, S. S. (1978). *Physical Review D*, **18**, 2745.

Bayin, S. S. (1979). *Physical Review D*, **19**, 2838.

Das, A. (1962). *Proceedings of the Royal Society A*, **267**, 1.

Das, A., and Agarwal, P. (1974). *General Relativity and Gravitation*, **5**, 359.

Dirac, P. A. M. (1938). *Proceedings of the Royal Society A*, **167**, 148.

Froyland, J. (1982). *Physical Review D*, **25**, 1470.

Gurses, M. (1977). *Physical Review D*, **15**, 2731.

Heller, M. (1982). *Acta Cosmologica Z*, **11**, 11.

Herrera, L., Jimenez, J., and Ruggeri, G. J. (1980). *Physical Review D*, **22**, 2305.

Hyde, J. M. (1963). *Proceedings of the Cambridge Philosophical Society*, **59**, 739.

Singh, K. M., and Bhamra, K. S. (1987). *International Journal of Theoretical Physics*, **26**, 175.

Tolman, R. C. (1934). *Relativity, Thermodynamics, and Cosmology*, Oxford University Press, New York.

Vaidya, P. C. (1951a). *Proceedings of the Indian Academy of Sciences A*, **33**, 264.

Vaidya, P. C. (1951b). *Physical Review*, **83**, 10.

Vaidya, P. C. (1966). *Astrophysical Journal*, **144**, 943.